

# Interacting Multiple Model (IMM) algorithm for road object tracking using automotive radar

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## Abstract:

Road object tracking is an integral part of automotive radar technology. In this paper, an Interacting Multiple Model (IMM) filter with three different dynamic models, has been formulated as a part of automotive radar tracking algorithm for tracking both maneuvering and non-maneuvering road objects. It has been shown that, this multiple model based tracking outperforms single filter based models, in all on-road traffic scenarios.

**Key words**—Radar tracking, Radar measurements, advanced driver assistance systems.

## 1. INTRODUCTION

In the modern day world of road fatalities Continental's radar is one of the world leaders in using automotive radar technology to save lives. Our ultimate goal is to ensure zero road fatality a reality and to achieve this, road object tracking (both maneuvering and non-maneuvering) plays a significant role in automotive tracking.

Road object tracking systems are one of the main fields of interest in the world of Intelligent Transport Systems by automotive radar. The key to successful target tracking lies in the optimal extraction of useful information about the target's state from the limited and noisy measurements obtained by automotive radar. Most tracking systems employ a single filter model to track maneuvering and non-maneuvering targets. But, single model based trackers do not perform well because model is often not matched to the target motion dynamics. In this context, multiple filter models enable a tracking system to better match changing target dynamics and thus overall tracking performance improves significantly for various on-road traffic scenarios.

Various mathematical models of target motion have been developed over the past three decades. For example – CV (Constant Velocity Model), CA (Constant Acceleration Model), CT (Coordinated Turn Rate Model), CTRV (Constant Turn Rate and Velocity Model), CTRA (Constant Turn Rate and Acceleration Model). These single filter models work well to track on road vehicles under specific scenarios e.g. CV model performs well when the target vehicle moves with constant velocity, but its performance starts to degrade as soon as the target vehicle accelerates or takes turn (in those cases CA

and CT model performs well respectively). In this paper, an Interacting Multiple Model (IMM) filter has been formulated for automotive radar tracking. IMM filter uses CV, CA and CTRV based target dynamics to model and estimate the entire vehicle trajectory.

## 2. SYSTEM MODEL

Linear Kalman Filter (KF) is used for time update and measurement update of CV-CV and CA-CV. For non-linear system (such as CTRV, CTRA models), we use Extended Kalman Filter (EKF) which approximates the posterior density as Gaussian by first order Taylor series linearization of nonlinear state transition and measurement function. The details of linear kalman filter and EKF can be found in [1].

### 2.1 Time Update or Predict:

We linearize state model about estimated state at k-1 (compute Jacobean) [1]

$$A_{k-1|k-1} = \frac{\partial F(x)}{\partial x} \Big|_{x=\hat{x}_{k-1|k-1}}$$

$$\hat{x}_{k|k-1} = F(\hat{x}_{k-1|k-1})$$

$$P_{k|k-1} = A_{k-1|k-1} P_{k-1|k-1} A_{k-1|k-1}^T + Q_k$$

### 2.2 Measurement Update:

We linearize measurement model about predicted state at k-1 (compute Jacobean)

$$C_{k|k-1} = \frac{\partial H(x)}{\partial x} \Big|_{x=\hat{x}_{k|k-1}}$$

The detailed algorithm can be found in [1].

EKF gives first order linearization of mean and covariance of the non-linear system. EKF is used time update and measurement update of CTRV model which is described later.

## 3. VEHICLE DYNAMIC MODEL

### 3.1. Constant Velocity (CV) Model:

The motion of a target vehicle can usually be modeled as moving by constant speed in straight. This is known as constant velocity (CV) model. For this model, the states under

consideration are:  $[x, \dot{x}, y, \dot{y}]$  where  $y$  corresponds to the lateral component,  $x$  corresponds to the longitudinal component,  $\dot{x}$  corresponds to velocity in x-direction,  $\dot{y}$  corresponds to velocity in y-direction. This model will yield the best estimates of position and velocity on non-maneuvering targets. We consider CV model for both lateral and longitudinal directions. For this model (which is actually CV-CV), state transition matrix

$$F_{CV} = \begin{bmatrix} 1 & dT & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & dT \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where,  $dT$  is the radar cycle time.

By CV-CV we mean, CV model is used in both x and y position estimates i.e. for both lateral and longitudinal directions.

The process noise covariance matrix  $Q$  of CV-CV filter is given by,

$$Q_{CV} = \begin{bmatrix} \alpha \frac{dT^4}{4} & \alpha \frac{dT^3}{2} & 0 & 0 \\ \alpha \frac{dT^3}{2} & \alpha dT^2 & 0 & 0 \\ 0 & 0 & \beta \frac{dT^4}{4} & \beta \frac{dT^3}{2} \\ 0 & 0 & \beta \frac{dT^3}{2} & \beta dT^2 \end{bmatrix}$$

Where  $\alpha$  and  $\beta$  are power spectral density (PSD) of process noise and they are used for tuning.

### 3.2. Constant Acceleration (CA) Model:

The motion of a target vehicle can usually be modeled as moving with acceleration in straight. This is known as constant velocity (CA) model. For this model, the states under consideration are:  $[x, \dot{x}, \ddot{x}, y, \dot{y}]$  where  $\ddot{x}$  corresponds to acceleration in x-direction. We consider CV model for lateral direction and CA model for longitudinal direction. For this model (which is actually CA-CV), state transition matrix is given by,

$$F_{CA} = \begin{bmatrix} 1 & dT & \frac{dT^2}{2} & 0 & 0 \\ 0 & 1 & dT & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & dT \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The process noise covariance matrix  $Q$  of CA-CV filter is given by,

$$Q_{CA} = \begin{bmatrix} \alpha \frac{dT^5}{20} & \alpha \frac{dT^4}{8} & \alpha \frac{dT^3}{6} & 0 & 0 \\ \alpha \frac{dT^4}{8} & \alpha \frac{dT^3}{3} & \alpha \frac{dT^2}{2} & 0 & 0 \\ \alpha \frac{dT^3}{6} & \alpha \frac{dT^2}{2} & \alpha dT & 0 & 0 \\ 0 & 0 & 0 & \beta \frac{dT^4}{4} & \beta \frac{dT^3}{2} \\ 0 & 0 & 0 & \beta \frac{dT^3}{2} & \beta dT^2 \end{bmatrix}$$

By CA-CV we mean, CA model is used in both x-position estimates (i.e. in lateral direction) and CV model is used in both y-position estimates (i.e. in longitudinal direction).

### 3.3. Constant Turn (CT) Model:

The motion of a vehicle can usually be modeled as moving by circle segments. This is coordinated/constant turn model and it is a non-linear model. For this model, the states under consideration are:  $[x, \dot{x}, y, \dot{y}, \omega]$  where  $\omega$  corresponds to turn rate. For this model state transition matrix,

$$F_{CT} = \begin{bmatrix} 1 & \frac{\sin(\omega t)}{\omega} & 0 & -\frac{1 - \cos(\omega t)}{\omega} & 0 \\ 0 & \cos(\omega t) & 0 & -\sin(\omega t) & 0 \\ 0 & \frac{1 - \cos(\omega t)}{\omega} & 1 & \frac{\sin(\omega t)}{\omega} & 0 \\ 0 & \sin(\omega t) & 0 & \cos(\omega t) & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The process noise covariance matrix  $Q$  of CTRV filter is given by,

$$Q_{CT} = \begin{bmatrix} \alpha \frac{dT^4}{4} & \alpha \frac{dT^3}{2} & 0 & 0 & 0 \\ \alpha \frac{dT^3}{2} & \alpha dT^2 & 0 & 0 & 0 \\ 0 & 0 & \beta \frac{dT^4}{4} & \beta \frac{dT^3}{2} & 0 \\ 0 & 0 & \beta \frac{dT^3}{2} & \beta dT^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\omega_{err}}^2 \end{bmatrix}$$

Where  $\sigma_{\omega_{err}}^2$  is the initial error in turn rate. This model assumes that the turn rate is known or could be estimated. When the range rate measurements are available, the turn rate could be estimated by using range rate measurements (i.e. radial velocity) [2], [3].

## 4. MEASUREMENT MODEL

In automotive radar, we get radar measurements of range, azimuth angles and radial velocity and their corresponding variances. We do not get the x-positions and y-positions directly. We convert these range, azimuth measurements from polar co-ordinate to Cartesian co-ordinate.

After this co-ordinate transformation, we get the measurements as follows:

$$x_m = x + \eta_x$$

$$y_m = y + \eta_y$$

$$\dot{r}_m = \dot{r} + \eta_r$$

$x$  = actual/true x-position of the target vehicle (relative to the ego vehicle)

$\eta_x, \eta_y, \eta_r$  = noise on x-position, y-position and radial velocity respectively. The measurement noise is assumed to white Gaussian as follows:

$$\eta_x \sim N(0, \sigma_x^2) \text{ and } \eta_y \sim N(0, \sigma_y^2).$$

$x_m$  = measured x-position of the target vehicle (relative to the ego vehicle)

$y_m$  = measured y-position of the target vehicle (relative to the ego vehicle)

$\dot{r}_m$  = measured radial velocity.

The variance of the x, y measurements  $[\sigma_x^2, \sigma_y^2]$  are indirectly obtained through range and azimuth measurement and their errors. Accordingly, the measurement covariance matrix  $R$  is calculated as follows:

$$R = \begin{bmatrix} R_{11} & R_{12} & 0 \\ R_{21} & R_{22} & 0 \\ 0 & 0 & R_{33} \end{bmatrix}$$

Where,

$$R_{11} = r_{err}^2 \sin^2(\theta_m) + r_m^2 \theta_{err}^2 \cos^2(\theta_m)$$

$$R_{22} = r_{err}^2 \cos^2(\theta_m) + r_m^2 \theta_{err}^2 \sin^2(\theta_m)$$

$$R_{12} = R_{21} = 0.5(\sin(2\theta_m)) (r_{err}^2 - r_m^2 \theta_{err}^2)$$

$R_{33}$  = variance of radial velocity and it is obtained directly as radar measurement. Here we neglect the co-relation between radial velocity measurement and x-y measurements.

The CV, CA and CT tracker use the measurements  $z_k = [x_m, y_m, \dot{r}]_k$  as inputs to it at time instant  $k$  and estimate the states (i.e. x-positions and y-positions). The final state estimate  $[\hat{x}, \hat{y}]_{k+1}$  is obtained by weighted fusion of state estimates of the three models. This is described in details, in the next section.

## 5. IMM TRACKER DESCRIPTION

The performance of a tracking system is governed by the performance of the state estimation algorithm employed. Accurate state estimation of targets in a tracking system is required for reliable data association and correlation. The states to be estimated are typically the kinematic quantities of position, velocity, and acceleration. Filters are used on measurements to reduce the uncertainty due to noise on the observation and to estimate quantities that are not directly observed. State estimation often require multiple filter models to account for varying target behaviors. IMM uses two or more

$y$  = actual/true y-position of the target vehicle (relative to the ego vehicle)

$\dot{r}$  = actual/true radial velocity and it's given by

$$\dot{r} = \frac{x\dot{x} + y\dot{y}}{\sqrt{x^2 + y^2}}$$

Kalman filters which run in parallel, each using a different model for target motion or errors. The IMM forms an optimal weighted sum of the output of all the filters and is able to rapidly adjust to target maneuvers. IMM considers all possible dynamics together – e.g. straight line motion, turn and motion with acceleration.

In our case, IMM combines state hypotheses from three dynamic models – CV, CA and CTRV. It automatically switches between different possible dynamics (e.g. CV to CTRV, or CTRV to CA etc) and gives prediction with high accuracy. In our case, IMM is a method for combining state hypotheses from three dynamic models – CV-CV, CA-CV and CTRV as shown in Figure 1. The entire algorithm is based on probabilistic homogeneous Markov chain. The individual filters switch among themselves based on their individual mode probabilities and the final fused state is estimated by weighting individual states from each model as shown in Figure 2.

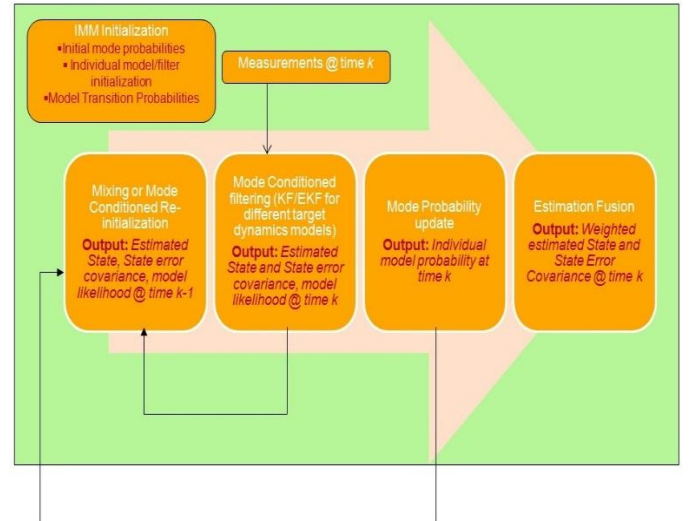


Figure -1: The Interacting Multiple Model (IMM): Basic processing blocks and Estimation Algorithm

The details of the IMM algorithm can be found in [4], [5] and [6]. One cycle IMM estimator steps are shown below in [7].

## 6. SIMULATION RESULTS

We have considered various test cases and compared the performance of single trackers (xy-coupled, xy-uncoupled, xy-separated, CTRV and CTVA where, xy-coupled, xy-uncoupled, xy-separated are special cases of CV-CV or CA-CV model) versus IMM tracker, for different target trajectories using fairly large number of Monte Carlo runs. Then we have plotted the RMS error of the estimates of the x-positions and y-positions for all the filters together in a single plot.



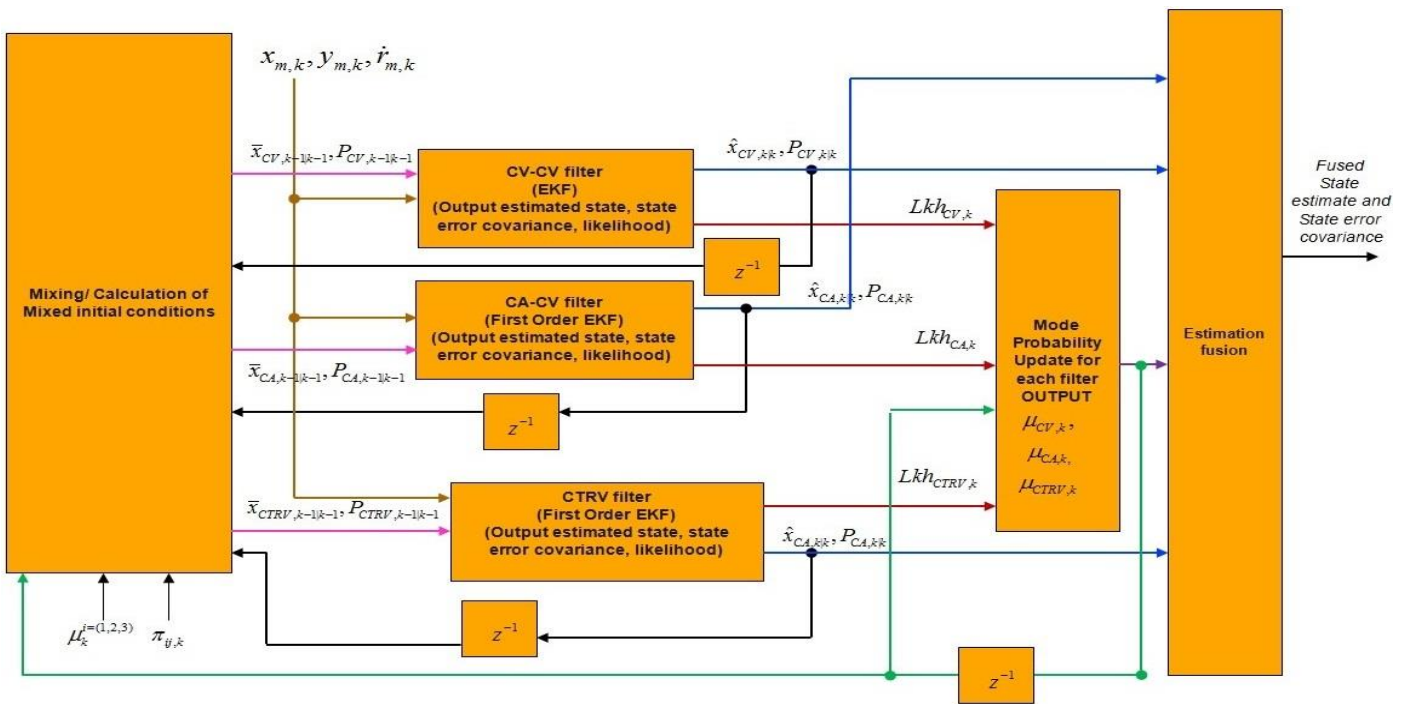


Figure -2: The Interacting Multiple Model (IMM): Mixing and Fused State Estimate

6.1. Ego Fixed, Target Vehicle – Sinusoidal maneuver:

In this case, we consider that ego vehicle (the vehicle on which the radar is mounted) is stationary and a target vehicle is moving in sinusoidal path inside the FOV (field-of-view) of the ego vehicle. The scenario is given in Figure 3 where the ego vehicle is assumed to be stationary at co-ordinate (0, 0). The RMS (root mean squared) error plots are given in Figure 4.

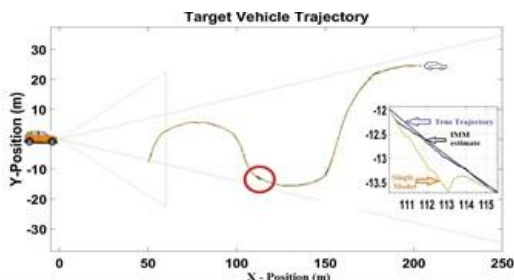


Figure -3: Improved tracking performance by IMM tracker (with three dynamic models – CV, CA, CTRV) compared to single CTRV model based tracker.

From Figure-4, we observe that, the RMS errors of estimated x-position and y-position for single trackers (e.g. xy-coupled, xy-uncoupled, xy-separated) can be as high as 1 meter and it clearly indicates track-loss by single trackers. The RMS error of IMM tracker is marginally less (in the range of approximately 2-4 cm) compared to all other single trackers. Thus IMM tracker provides improved tracking performance.

6.2. Ego moving, Target Vehicle – semi-circular maneuver:

In this case, we consider that ego vehicle is moving with constant velocity and a target vehicle is moving in semi-circular path inside the field-of-view of the ego vehicle. The scenario and the RMS error plots are given in Figure 5 and Figure 6

respectively. In Figure-5, the FOV is shifting towards x-direction because of the longitudinal ego motion.

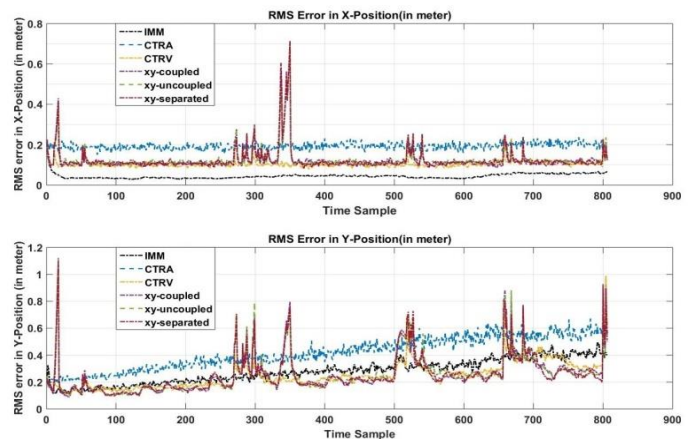


Figure -4: RMS Error (in m) of estimated x-position and estimated y-position

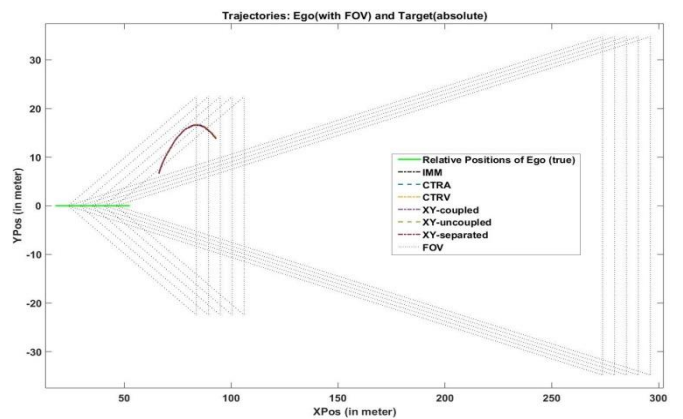


Figure -5: Trajectories of ego vehicle and target vehicle (in absolute frame)

From Figure 6, we observe that, estimation error in IMM less than that of single trackers.

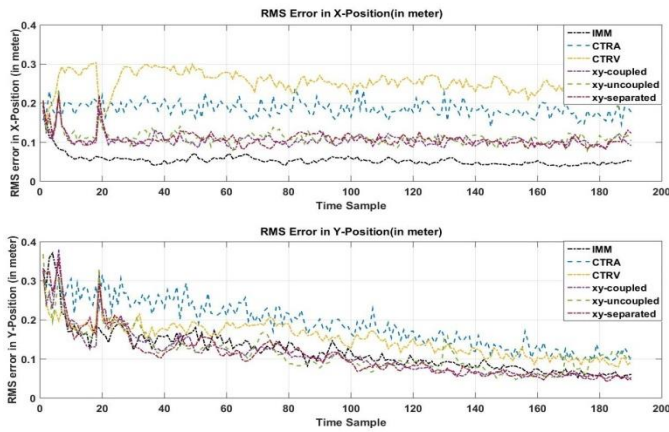


Figure -6: RMS Error (in m) of estimated x-position and estimated y-position

6.3. Ego stationary, Target Vehicle – U-turn:

In this case, we consider that Ego vehicle is stationary and a target vehicle is taking a U-turn inside the field-of-view of the ego vehicle. The scenario and the RMS error plots are given in Figure 7 and Figure 8 respectively.

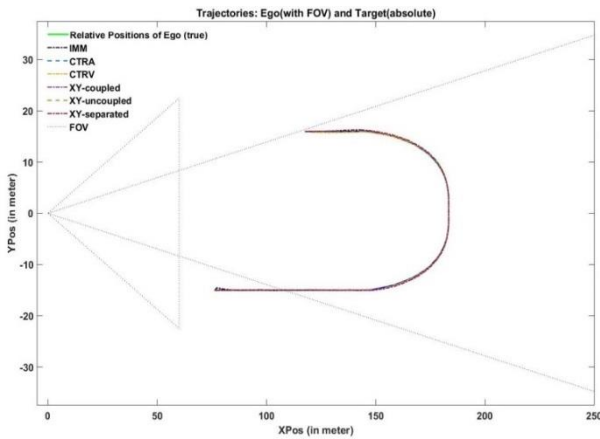


Figure -7: Trajectories of ego vehicle and target vehicle (in absolute frame)

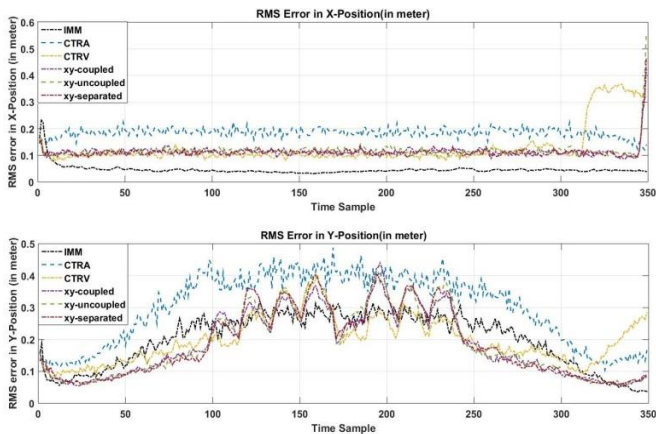


Figure -8: RMS Error (in m) of estimated x-position and estimated y-position

From Figure-8, we observe that, the RMS error of x-y position estimate of IMM tracker is around 1-5 cm, which is quite good and reasonable. But, for single trackers it's can be as high as 0.6 meter, which indicates very poor performance by single trackers.

CONCLUSION

In this paper, we have shown that we need switch from traditional single trackers to multiple tracker models to improve overall tracking performance, especially for turning and changing maneuver. We have also shown that interacting multiple model or IMM tracker (under Extended Kalman Filter-EKF set-up) which internally switches between three dynamic models (namely, CV, CA and CTRV) as per probabilistic Markov Chain, has superior performance than any standard single tracker, in all real-life traffic scenarios.

REFERENCES

[1] Hartikainen, Jouni, Arno Solin, and Simo Särkkä. "Optimal filtering with Kalman filters and smoothers." *Department of Biomedical Engineering and Computational Sciences, Aalto University School of Science, Greater Helsinki, Finland* 16, 2011.

[2] Frencl, Victor B., and Joao BR do Val. "Tracking with range rate measurements: Turn rate estimation and particle filtering." *In Radar Conference (RADAR), IEEE* 2012, pp. 0287-0292.

[3] Yuan, Xianghui, Chongzhao Han, Zhansheng Duan, and Ming Lei. "Adaptive turn rate estimation using range rate measurements." *IEEE Transactions on Aerospace and Electronic Systems*, vol 42, no. 4, 2006.

[4] Mazor, Efim, Amir Averbuch, Yakov Bar-Shalom, and Joshua Dayan. "Interacting multiple model methods in target tracking: a survey." *IEEE Transactions on aerospace and electronic systems*, vol 34, no. 1, 1998, pp 103-123.

[5] Lee, Sangjin, and Inseok Hwang. "Interacting multiple model estimation for spacecraft maneuver detection and characterization." *In AIAA Guidance, Navigation, and Control Conference*, 2015, pp. 2015-1333.

[6] Genovese, Anthony F. "The interacting multiple model algorithm for accurate state estimation of maneuvering targets." *Johns Hopkins APL technical digest*, vol 22, no. 4, 2001, pp 614-623.

[7] Li, X. Rong, and Vesselin P. Jilkov. "Survey of maneuvering target tracking. Part V. Multiple-model methods." *IEEE Transactions on Aerospace and Electronic Systems*, vol 41, no. 4, 2005, pp 1255-1321.

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